定理：若 $2\nmid r$，$1^r+2^r+\cdots+k^r$ 被 $k(k+1)$ 整除

复杂知识：配对、整数、整体处理

（求值）

题. 设 $n\in\mathbb{N}_+$，求证：$512\mid 3^{2n}-32n^2+24n-1$.

证明：设 $f(n)=3^{2n}-32n^2+24n-1$

$512\mid f(n) \iff 512\mid f(n+1)-f(n)$ 且 $512\mid f(1)=0$ 成立

记 $g(n)=f(n+1)-f(n)=3^{2n+2}-32(n+1)^2-24(n+1)-1-3^{2n}+32n^2-24n+1$

$=9\cdot 3^{2n}-32n^2-64n-32+24n+24-1-3^{2n}$

$=8\cdot 3^{2n}-64n-8$

$=8(3^{2n}-8n-1)$

欲证 $512\mid g(n)=8(3^{2n}-8n-1)$，只需证 $64\mid 3^{2n}-8n-1$

记 $h(n)=3^{2n}-8n-1$

$\therefore h(n+1)-h(n)=9\cdot 3^{2n}-8n-9-3^{2n}+8n+1$

$=8\cdot 3^{2n}-8$

$\therefore \cdots$，只要证 $8\mid 3^{2n}-1$

令 $a(n)=3^{2n}-1$ $\therefore a(n+1)-a(n)=3\cdot 3^{2n}-1+3^{2n}+1=4^{2n}=16^n$

$\therefore 8\mid 16^n$ $\therefore$ 原命题得证

题. 设 p 和 q 为正整数，满足

$$\frac{p}{q}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots-\frac{1}{1318}+\frac{1}{1319}$$

求证：p 可被 1979 整除.

证明：$\frac{p}{q}=\left(1+\frac{1}{2}+\cdots+\frac{1}{1319}\right)-2\left(\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{1318}\right)$

$=\frac{1}{660}+\frac{1}{661}+\cdots+\frac{1}{1318}+\frac{1}{1319}$

$=\left(\frac{1}{660}+\frac{1}{1319}\right)+\left(\frac{1}{661}+\frac{1}{1318}\right)+\cdots+\left(\frac{1}{989}+\frac{1}{990}\right)$

$=\underbrace{\left(\frac{1}{660\times 1319}+\frac{1}{661\times 1318}+\cdots+\frac{1}{989\times 990}\right)}_{①}\times 1979$

$\because$ 1979 为质数 $\therefore$ 660 ~ 990 与 1979 最大公因数均为 1

$\therefore$ 设 $660\times 661\times\cdots\times 1319=n$，$660+\cdots+1319=m$

$\therefore \frac{p}{q}=\frac{1979m}{n}$，$\gcd(n,1979)=1$

$\therefore 1979\mid p$