第一次讲义：

Ex 12.7

解：$\because 120 < 11 \times 11$ $\therefore$ 120以内所有合数均为2、3、5、7的倍数

2的倍数：$[\frac{120}{2}] = 60$，3的倍数：$[\frac{120}{3}] = 40$，
5的倍数：$[\frac{120}{5}] = 24$，7的倍数：$[\frac{120}{7}] = 17$

合数：$[\frac{120}{2}] + [\frac{120}{3}] + [\frac{120}{5}] + [\frac{120}{7}] - [\frac{120}{6}] - [\frac{120}{10}] - [\frac{120}{14}] - [\frac{120}{15}]$

$- [\frac{120}{21}] - [\frac{120}{35}] + [\frac{120}{30}] + [\frac{120}{42}] + [\frac{120}{70}] + [\frac{120}{105}] - \underline{4} \rightarrow$ 2,3,5,7本身

$= 60 + 40 + 24 + 17 - 20 - 12 - 8 - 8 - 5 - 3 + 4 + 2 + 1 + 1 - 4$

$= 93 - 4 = 89$ 个

素数：$120 - 93 + 4 - 1 = 30$ 个

Ex. 1.2.8

解：显然。根据容斥原理，有 $3^n - C_3^2 \cdot 2^n + C_3^1 \cdot 1^n = 3^n - 3 \times 2^n + 3$

Ex. 1.2.9

解：i $\begin{cases} x_1 + x_2 = -p-2 \\ x_1 x_2 = 1 \end{cases}$ 且 $\Delta = (p+2)^2 - 4 \geq 0$ ii $\Delta = (p+2)^2 - 4 < 0$

$\because A = \{x_1, x_2\} \subset R^-$ $\therefore -4 < p < 0$

$\therefore -p-2 < 0$，$p > -2$ $\therefore p \geq 0$ 此时 $A = \varnothing$

$\therefore$ 综上，$p \in (-4, +\infty)$

Ex. 1.2.10

证明：若其为有理数，设 $\sqrt{2} = \frac{p}{q}$，p.q互素

$\therefore p^2 = 2q^2$

若p.q同为奇数，则上式一定不成立
若一奇一偶，则p为偶，q为奇 $\therefore 4 | p^2$ $\therefore 2 | q^2$，矛盾
若p.q同偶，则与p.q互素矛盾

$\therefore$ 假设不成立。 $\therefore \sqrt{2}$为无理数

Ex. 1.2.11

Ran women xian lai zuo xie changshi:

$|A| = 1$，$\rightarrow$ $\varnothing$，$\{a_1\}$，
$|A| = 2$ $\rightarrow$ $\varnothing$，$\{a_1\}$，| $\{a_1, a_2\}$，$\{a_2\}$
$|A| = 3$ $\rightarrow$ $\varnothing$，$\{a_1\}$，$\{a_1, a_2\}$，$\{a_2\}$，| $\{a_2, a_3\}$，$\{a_1, a_2, a_3\}$，$\{a_1, a_3\}$，$\{a_3\}$

Duiyu $|A| = n+1$，zhiyao da $|A| = n$ de qingkuang daoguolai，ranhou dou jiashang $\{a_{n+1}\}$

$\therefore$ 证明：对于 $n=1$，显然成立

对于 * $n = k$，假设存在 $S_1, S_2, \ldots, S_m$ 满足情况，$S_i \subseteq A_k$，

$A_k \subseteq A_{k+1}$ 且 $A_{k+1} \setminus A_k = \{a_{k+1}\}$ 取逆序数再并上$\{a_{k+1}\}$

则 对于 $n = k+1$，集合列 $S_1, S_2, \ldots, S_m$，| $S_m \cup \{a_{k+1}\}$，$S_{m-1} \cup \{a_{k+1}\}$，$\ldots$ $S_1 \cup \{a_{k+1}\}$ 满足情况

$3x = y-3$

$4-x$ 无关内容

$1-x$

$x = \frac{1}{4}$, $y = 4 - \frac{1}{4} = \frac{15}{4}$

Ex. 1.2.14

解：$\because M = \{x, xy, \lg xy\}$

$N = \{0, |x|, y\}$

$\therefore x = -1,\ y = -1$

$\therefore \sum_{n=1}^{2015} \left(x^n + \frac{1}{y^n}\right) = -2$

Ex. 1.2.15

解：Trivial.

Ex. 1.2.16

解：猜测：只有唯一的公共元？

断言：$\exists a \in A_1$，且$\exists$其它45个集合，都含有a

$\rightarrow$ 反面：$\forall b \in A_i$，至少有44个集合含有b

$\because 44 \times 45 = 1980 < 1999$

$\therefore$ 否命题为假 $\therefore$ 断言为真

$\therefore$ 令含有a的集合为$A_1, A_2, \cdots A_{46}$，如果有$B = A_{47} \sim A_{1999}$，$a \notin B$

下面推出矛盾：

设 $B \cap A_1 = \{b_1\}$, $B \cap A_2 = \{b_2\}$, ……, $b \cap A_{46} = \{b_{46}\}$

$\because a \notin B$ $\therefore b_1 \neq b_2 \neq \cdots \neq b_{46}$ $\therefore |B| \geq 46$，与$|B| = 45$矛盾

$\therefore a \in A_{47} \sim A_{1999}$

$\therefore |A_1 \cup A_2 \cup \cdots \cup A_{1999}| = 45 \times 1999 - 1 = 90000 - 46 = 89954$